

Code-Based Cryptography

1. Error-Correcting Codes and Cryptography
2. McEliece Cryptosystem
3. Message Attacks (ISD)
4. **Key Attacks**
5. Other Cryptographic Constructions Relying on Coding Theory

4. Key Attacks

1. Introduction
2. Support Splitting Algorithm
3. Distinguisher for GRS codes
4. Attack against subcodes of GRS codes
5. **Error-Correcting Pairs**
6. Attack against GRS codes
7. Attack against Reed-Muller codes
8. Attack against Algebraic Geometry codes
9. Goppa codes still resist

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An efficient decoding algorithm for GRS codes - ECP

Let $\mathcal{C} = \text{GRS}_k(\mathbf{c}, \mathbf{d}) \implies \mathcal{C}^\perp = \text{GRS}_{n-k}(\mathbf{c}, \mathbf{d}^\perp)$

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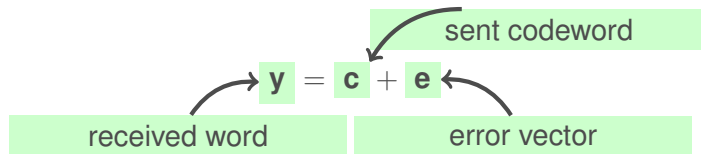
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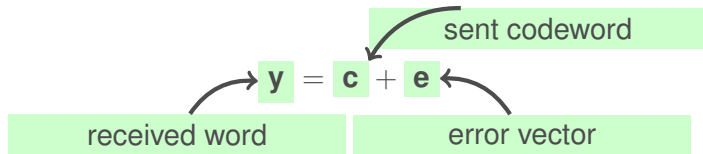
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Define:

$$K_{\mathbf{y}} = \left\{ \mathbf{a} \in \mathcal{A} \mid \langle \mathbf{y}, \mathbf{a} * \mathbf{b} \rangle = 0, \text{ for all } \mathbf{b} \in \mathcal{B} \right\}$$

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Thus, for all $\mathbf{a} \in \mathcal{A}$ and $\mathbf{b} \in \mathcal{B}$

$$\langle \mathbf{y}, \mathbf{a} * \mathbf{b} \rangle$$

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If there have been no more than t errors (i.e. $w_H(\mathbf{e}) \leq t$), then \mathbf{e} is a solution of:

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which **contradicts** the minimality of $d(\mathcal{C})$.

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- **AG** codes also have a t -ECP
- ECP for **cyclic codes** were investigated by Duursma and Kötter.



I. Duursma

Decoding codes from curves and cyclic codes.

Ph.D thesis, Eindhoven University of Technology (1993)



I. Duursma, R. Kötter.

Error-locating pairs for cyclic codes.

IEEE Trans. Inform. Theory, Vol.40, 1108–1121 (1994)

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On decoding by error location and dependent sets of error positions.

Discrete Math., 106–107: 369–381 (1992).



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A unified description of an error locating procedure for linear codes.

In Proceedings of Algebraic and Combinatorial Coding Theory, 113–117. Voneshta Voda (1992).

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Let:

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→ A be an $[n, K(A)]_{q^m}$ code

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(A, B) is a **t -ECP** for \mathcal{C} if the following properties hold:

E.1 $(A * B) \perp \mathcal{C}$.

E.2 $K(A) > t$.

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E.4 $d(A) + d(\mathcal{C}) > n$.



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An $[n, k]_q$ code which has a t -ECP over \mathbb{F}_{q^m} has an efficient decoding algorithm.



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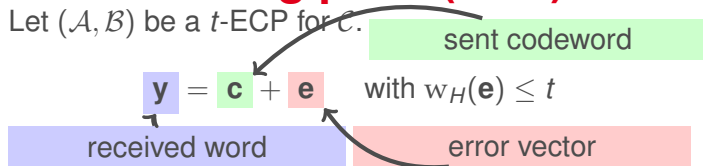
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Let $(\mathcal{A}, \mathcal{B})$ be a t -ECP for \mathcal{C} .



1. There exists $\mathbf{a} \in \mathcal{A}$, $\mathbf{a} \neq \mathbf{0}$ such that

$$\langle \mathbf{y}, \mathbf{a} * \mathbf{b} \rangle = 0 \text{ for all } \mathbf{b} \in \mathcal{B} \quad (1)$$

2. For every solution $\mathbf{a} \in \mathcal{A}$ of (1) we have that:

$$\mathbf{a} * \mathbf{e} = \mathbf{0}$$

3. Since $d(\mathcal{A}) + d(\mathcal{C}) \geq n$. Then, \mathbf{e} is the **unique** solution of:

$$\langle \mathbf{e}, \mathbf{a} * \mathbf{b} \rangle = 0 \text{ with } \mathbf{e} * \mathbf{a} = 0 \text{ for all } \mathbf{b} \in \mathcal{B}$$

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