

# Code-Based Cryptography

Error-Correcting Codes and Cryptography

# 1. Error-Correcting Codes and Cryptography

1. Introduction I - Cryptography
2. **Introduction II - Coding Theory**
3. Encoding (Linear Transformation)
4. Parity Checking
5. Error Correcting Capacity
6. Decoding (A Difficult Problem)
7. Reed-Solomon Codes
8. Goppa Codes
9. McEliece Cryptosystem

# Introduction to Coding Theory

*“A Mathematical Theory of Communication”*  
(Claude Shannon, 1948)

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graph TD; A["A Mathematical Theory of Communication (Claude Shannon, 1948)"] --> B["Coding Theory"]; A --> C["Information Theory"];
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The diagram illustrates the foundational role of Claude Shannon's 1948 paper, "A Mathematical Theory of Communication," in the development of Coding Theory and Information Theory. A central grey box at the top contains the title and author. Two curved arrows point from this box to two separate red boxes below, labeled "Coding Theory" and "Information Theory," indicating that the paper's work underpins both fields.

**Coding Theory**

**Information Theory**

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**Coding Theory**

**Information Theory**

*"Efficient transfer reliable information"*

# Communication System

Alphabet:  $\mathcal{A}$



# Communication System

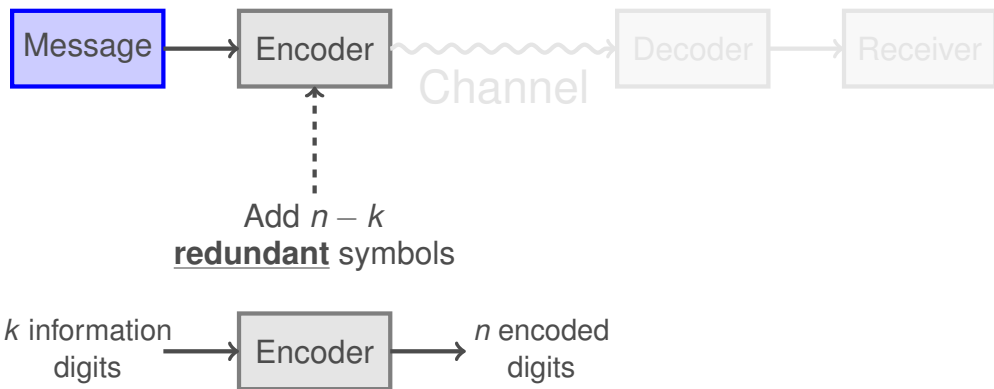
Alphabet:  $\mathcal{A}$



$$\mathbf{m} = (m_1 \dots, m_k) \in \mathcal{A}^k$$

# Communication System

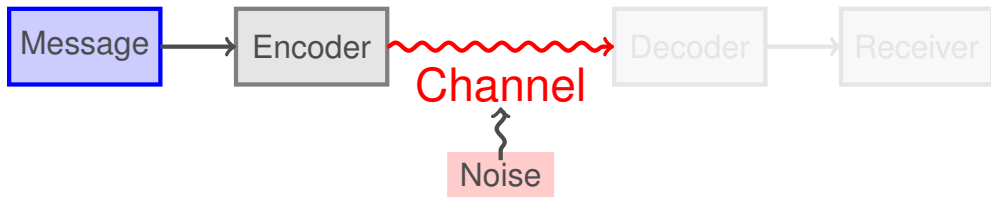
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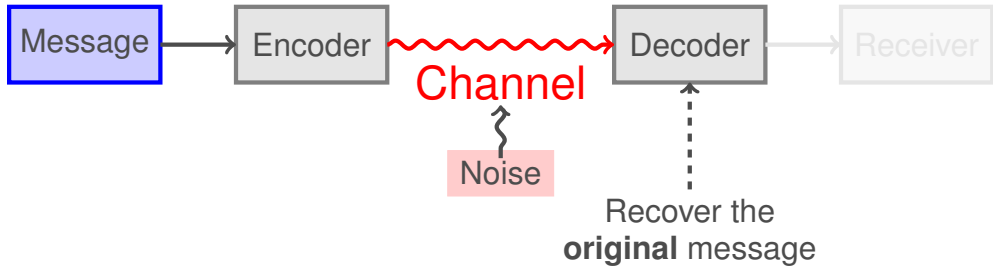
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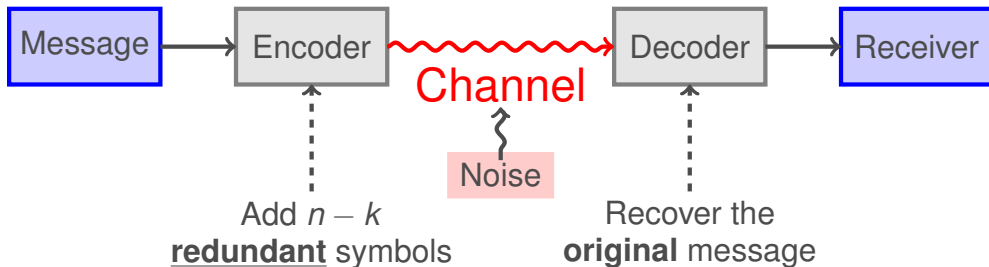
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$$\text{Decoder} \left( \underbrace{\text{Encoder} (\text{Message})}_{\text{codeword}} + \text{Noise} \right) = \text{Message}$$

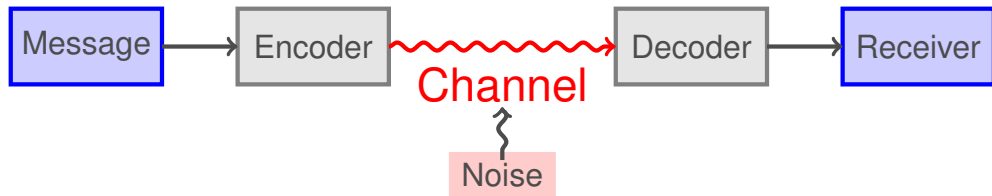
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# Communication System - Repetition code



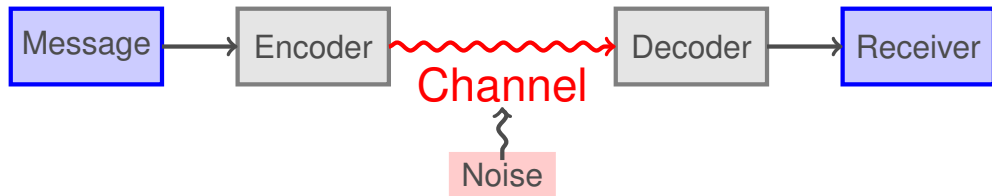
1. **Alphabet:**  $\mathcal{A}$  = English Alphabet

Yes

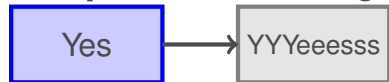
2. **Alphabet:**  $\mathcal{A} = \mathbb{F}_2$

1

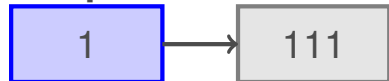
# Communication System - Repetition code



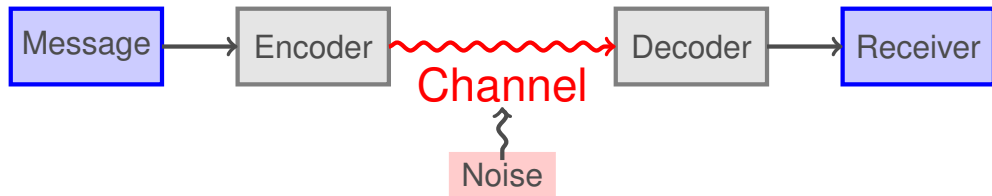
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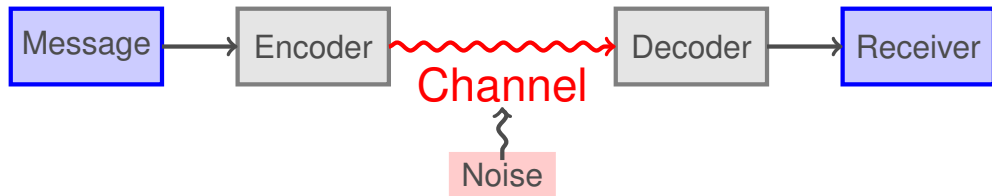
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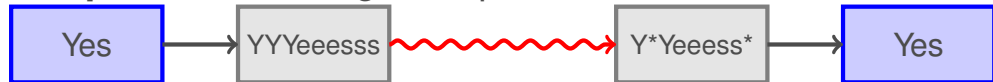
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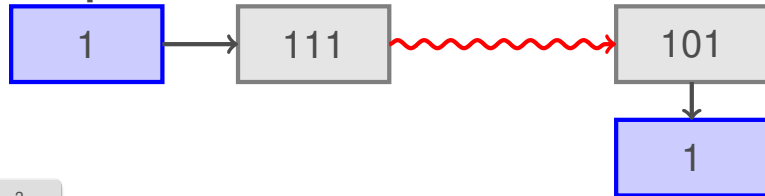
# Communication System - Repetition code



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# Claude Shannon





# Claude Shannon



→ Definition of Information

# Claude Shannon



→ Definition of Information

→ “Channel Coding Theorems”

# Claude Shannon



→ Definition of Information

→ “Channel Coding Theorems”

“For any communication channel it is possible to communicate discrete data nearly error free up to a maximum rate (**channel capacity**)”

# Check Digit

A **check digit** is a redundancy bit used for detecting one error

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## International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.  
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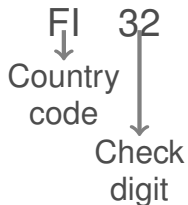
FI  
↓  
Country  
code

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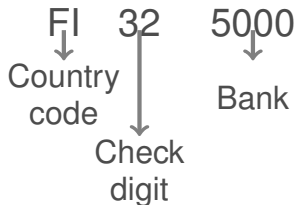


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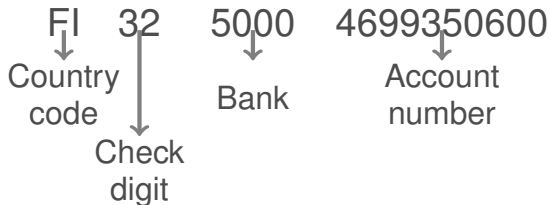


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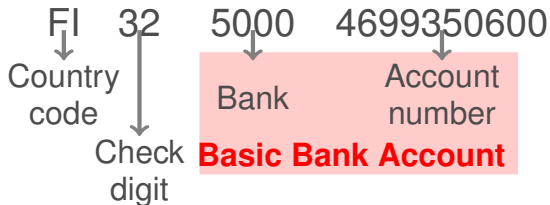


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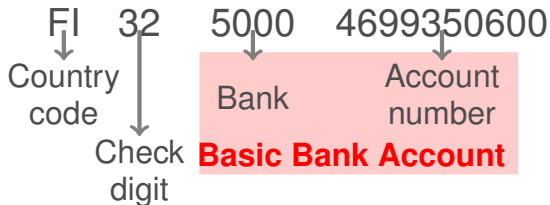


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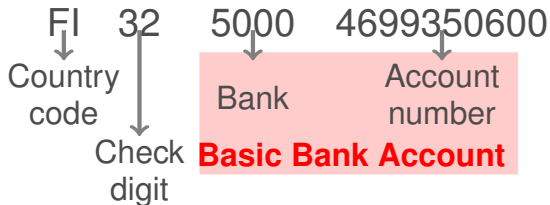
**FI**32 5000 4699350600

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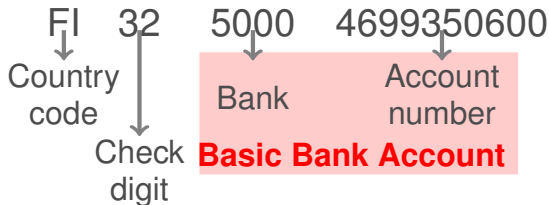
### 1. Rearrange

# Check Digit

A **check digit** is a redundancy bit used for detecting one error

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32 5000 4699350600 **1518**

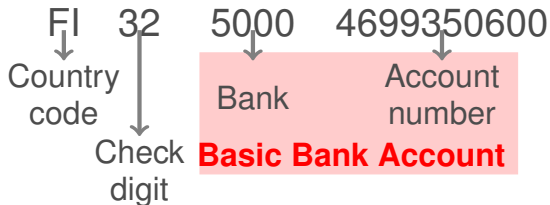
1. Rearrange
2. Convert to Integer

# Check Digit

A **check digit** is a redundancy bit used for detecting one error

## International Bank Account Number (IBAN)

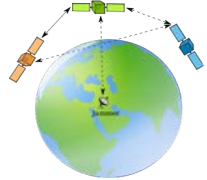
The **IBAN** is used for identifying bank account across national borders.  
It consist of 4 elements:



$$32\ 5000\ 4699350600\mathbf{1518} \equiv 1 \pmod{97}$$

1. Rearrange
2. Convert to Integer
3. Validate

# Error correcting codes in our daily life



# Coding Theory vs. Cryptography

**Coding Theory**

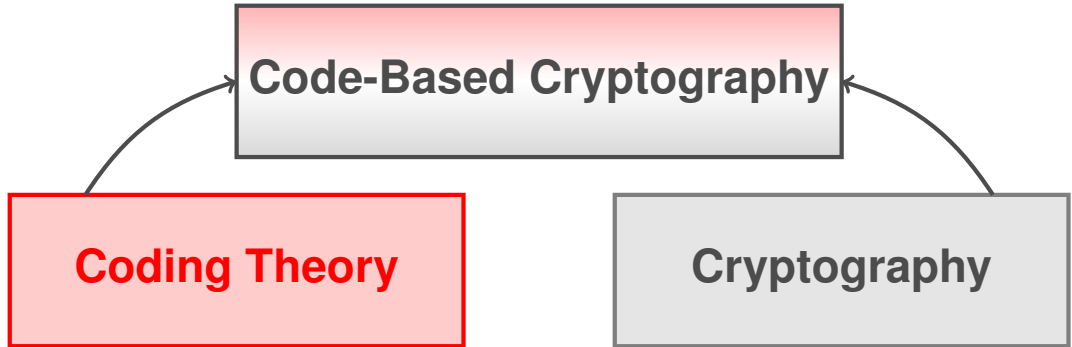


# Coding Theory vs. Cryptography

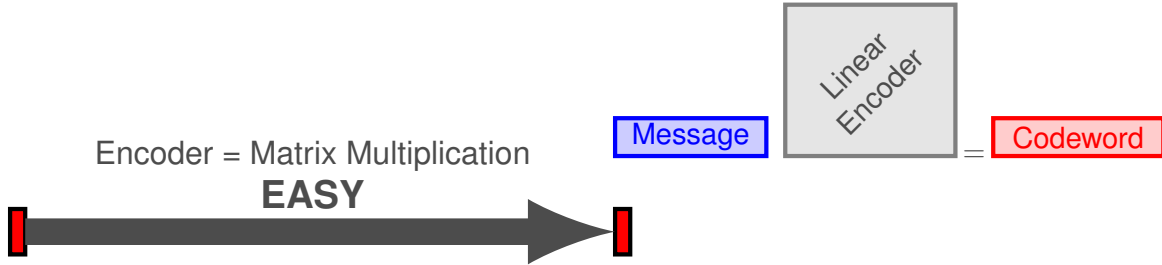
**Coding Theory**

**Cryptography**

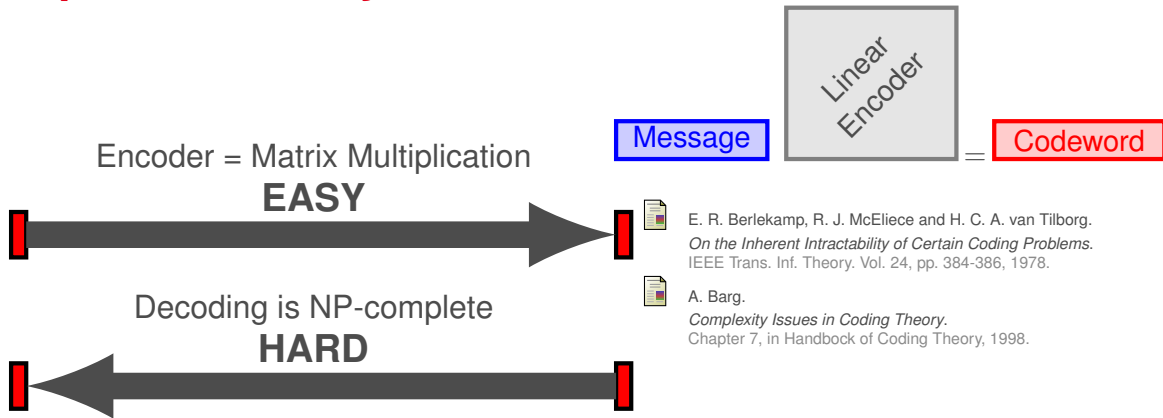
# Coding Theory vs. Cryptography



# Trapdoor one-way functions - Decoder



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# Trapdoor one-way functions - Decoder

Encoder = Matrix Multiplication

**EASY**

Message

Linear  
Encoder

Codeword

Decoding is NP-complete

**HARD**

Efficient decoder for certain families of codes

**EASY** (with TRAPDOOR information)

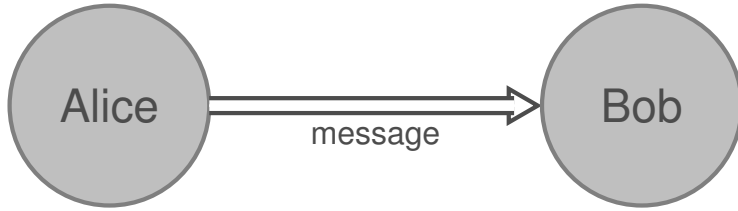


E. R. Berlekamp, R. J. McEliece and H. C. A. van Tilborg.  
*On the Inherent Intractability of Certain Coding Problems.*  
IEEE Trans. Inf. Theory. Vol. 24, pp. 384-386, 1978.



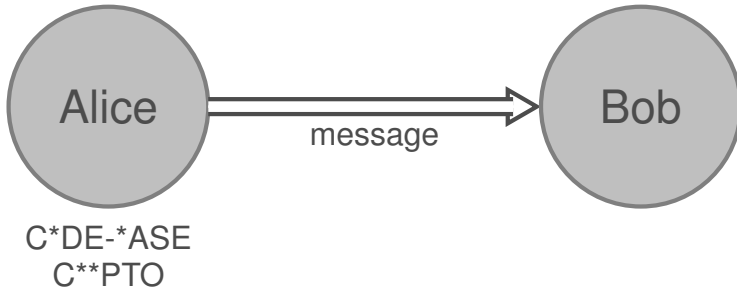
A. Barg.  
*Complexity Issues in Coding Theory.*  
Chapter 7, in Handbook of Coding Theory, 1998.

# How to use Coding Theory in Cryptography?



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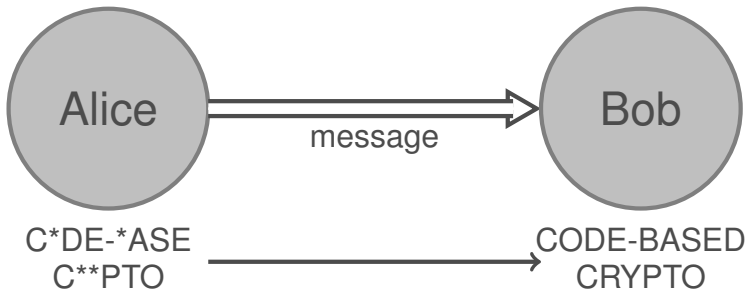
Adds errors  
in the message



# How to use Coding Theory in Cryptography?

Adds errors  
in the message

Knows an efficient  
decoding method





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