Code-Based Cryptography

Error-Correcting Codes and Cryptography

I. Márquez-Corbella

informatiques mathématiques

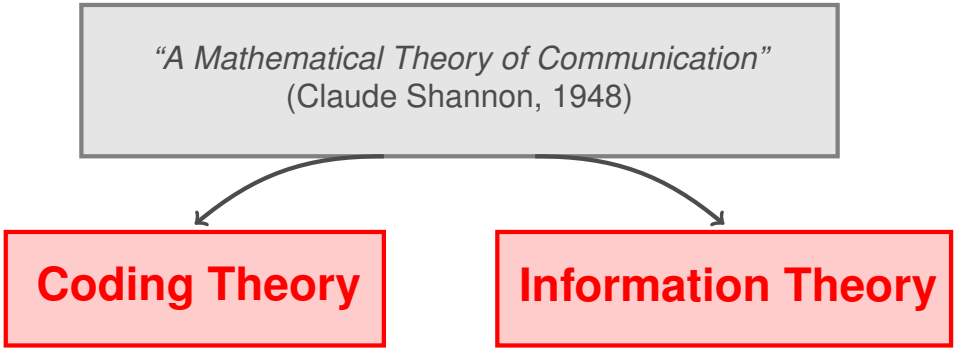
1. Error-Correcting Codes and Cryptography

1. Introduction I - Cryptography
2. **Introduction II - Coding Theory**
3. Encoding (Linear Transformation)
4. Parity Checking
5. Error Correcting Capacity
6. Decoding (A Difficult Problem)
7. Reed-Solomon Codes
8. Goppa Codes
9. McEliece Cryptosystem

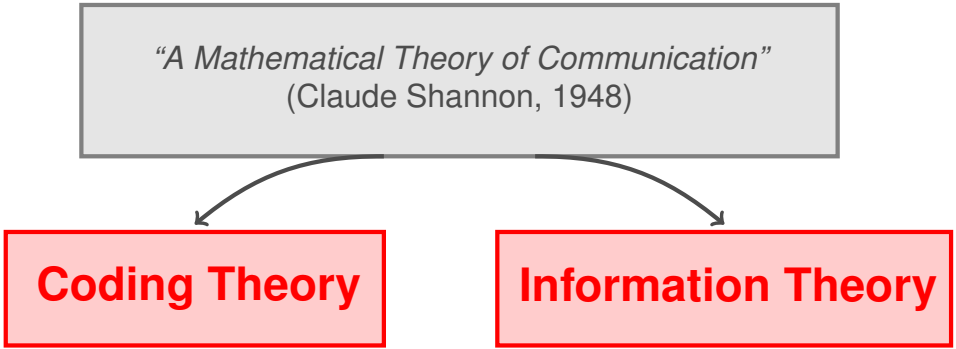
Introduction to Coding Theory

“A Mathematical Theory of Communication”
(Claude Shannon, 1948)

Introduction to Coding Theory



Introduction to Coding Theory



" *Efficient* *transfer* *reliable* *information*"

Communication System

Alphabet: $\mathcal{A}$

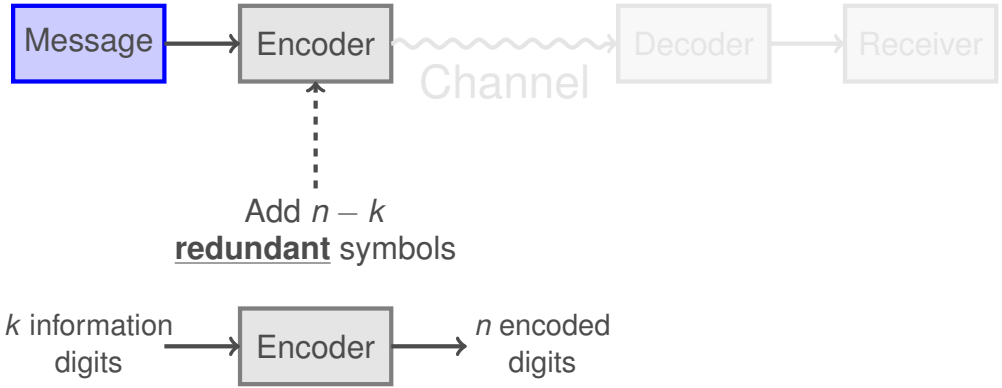
Communication System

Alphabet: $\mathcal{A}$

$\mathbf{m} = (m_1 \ldots, m_k) \in \mathcal{A}^k$

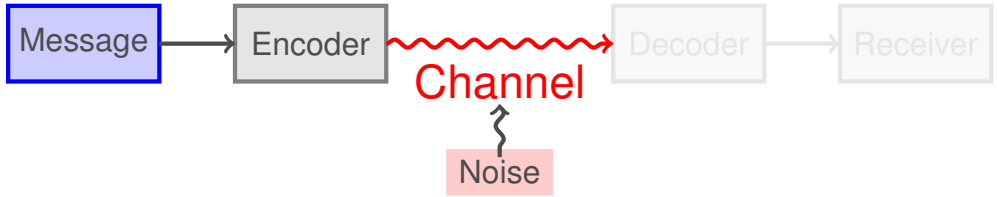
Communication System

Alphabet: $\mathcal{A}$



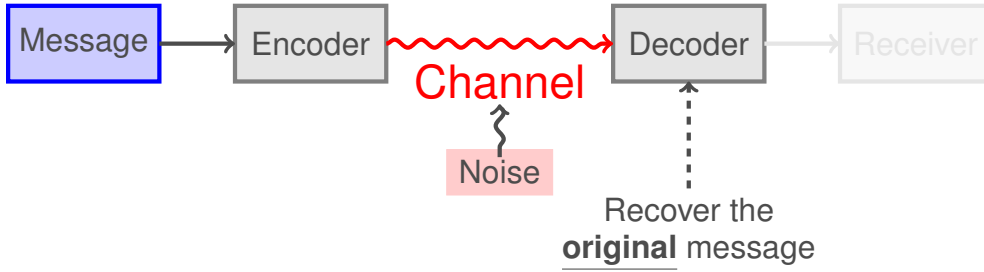
Communication System

Alphabet: $\mathcal{A}$



Communication System

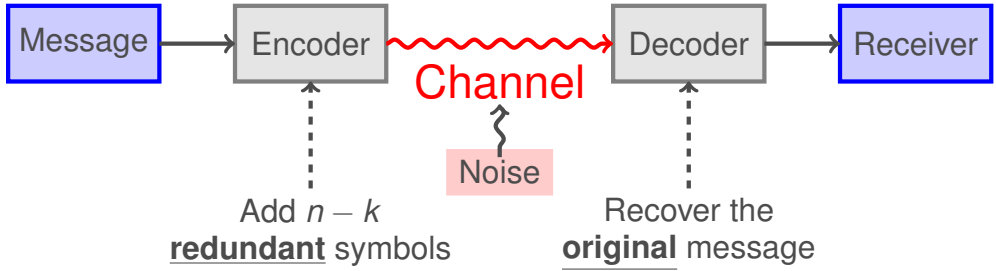
Alphabet: $\mathcal{A}$



$$\texttt{Decoder}\big(\underbrace{\texttt{Encoder}\big(\texttt{Message}\big)}_{\text{codeword}} + \texttt{Noise}\big) = \texttt{Message}$$

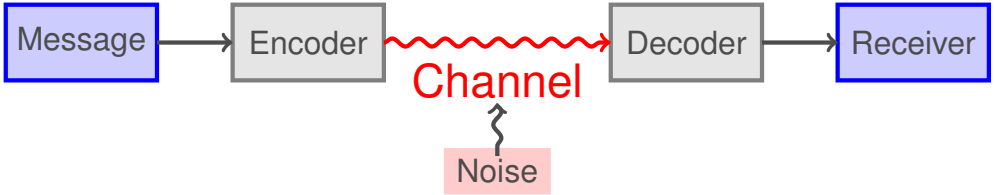
Communication System

Alphabet: $\mathcal{A}$



$$\texttt{Decoder}\big(\underbrace{\texttt{Encoder}\big(\texttt{Message}\big)}_{\texttt{codeword}} + \texttt{Noise}\big) = \texttt{Message}$$

Communication System - Repetition code



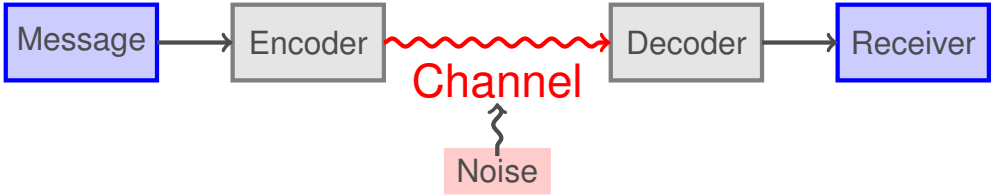
1. **Alphabet**: $\mathcal{A}$ = English Alphabet

Yes

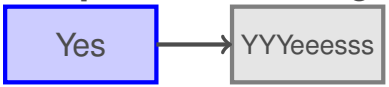
2. **Alphabet**: $\mathcal{A} = \mathbb{F}_2$

1

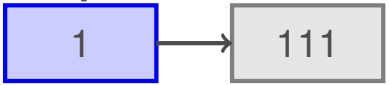
Communication System - Repetition code



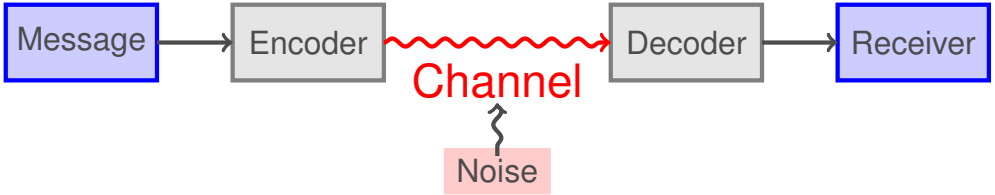
1. **Alphabet**: $\mathcal{A}$ = English Alphabet



2. **Alphabet**: $\mathcal{A} = \mathbb{F}_2$

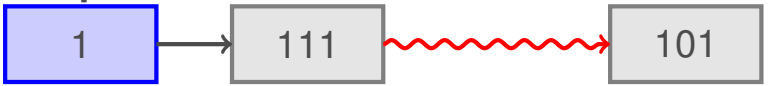


Communication System - Repetition code

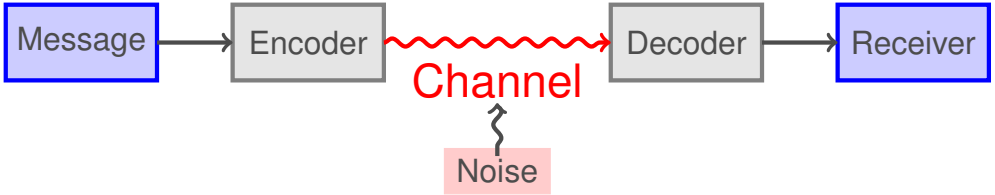


1. **Alphabet**: $\mathcal{A}$ = English Alphabet

2. **Alphabet**: $\mathcal{A} = \mathbb{F}_2$

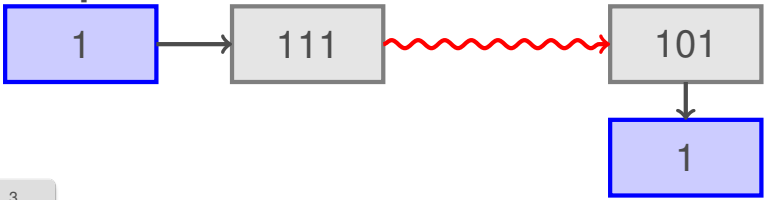


Communication System - Repetition code

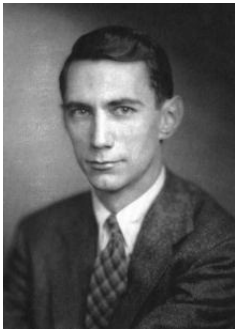


1. **Alphabet**: $\mathcal{A}$ = English Alphabet

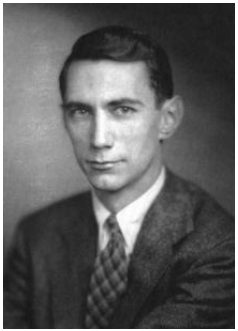
2. **Alphabet**: $\mathcal{A} = \mathbb{F}_2$



Claude Shannon

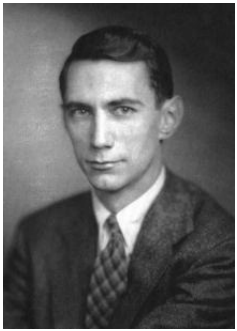


Claude Shannon



➜ Definition of Information

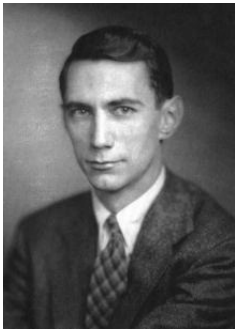
Claude Shannon



➜ Definition of Information

➜ “Channel Coding Theorems”

Claude Shannon



➜ Definition of Information

➜ “Channel Coding Theorems”

“For any communication channel it is possible to communicate discrete data nearly error free up to a maximum rate **(channel capacity)**”

Check Digit

A **check digit** is a redundancy bit used for detecting one error

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

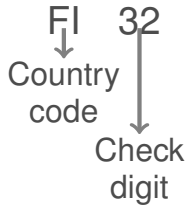
The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

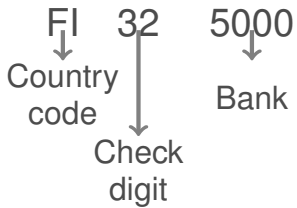


Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

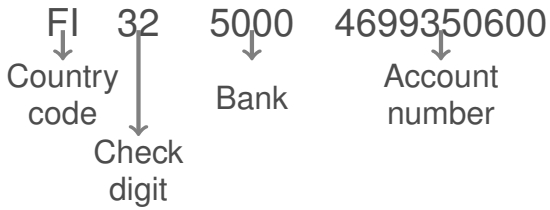


Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

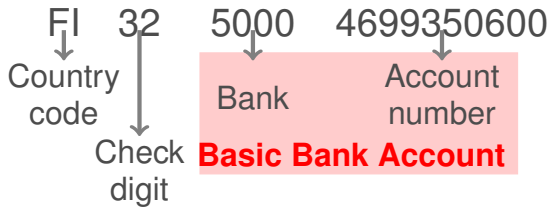


Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

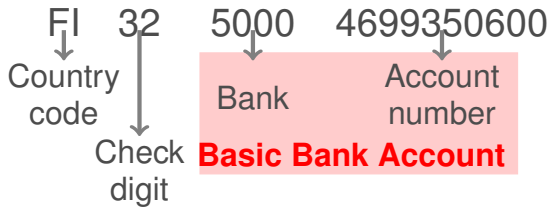


Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:



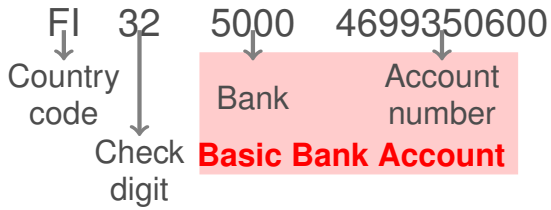
FI 32 5000 4699350600

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:



32 5000 4699350600 FI

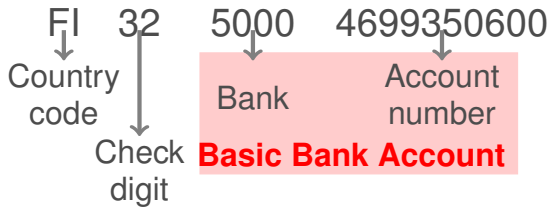
1. **Rearrange**

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:



32 5000 469935060015 18

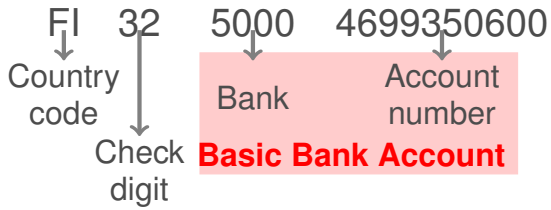
1. **Rearrange** 2. **Convert to Integer**

Check Digit

A **check digit** is a redundancy bit used for detecting one error

International Bank Account Number (IBAN)

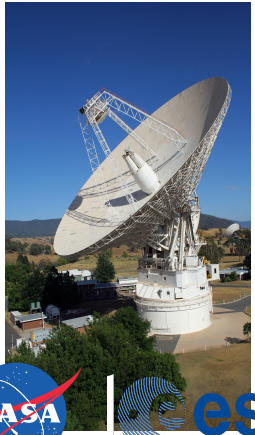
The **IBAN** is used for identifying bank account across national borders.
It consist of 4 elements:

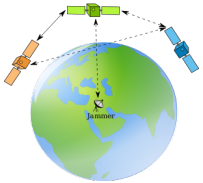


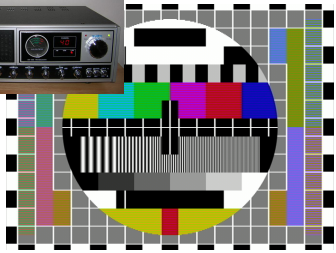
$$32\,5000\,469935060015\,18 \equiv 1 \mod 97$$

1. **Rearrange**
2. **Convert to Integer**
3. **Validate**

Error correcting codes in our daily life







Coding Theory vs. Cryptography

Coding Theory

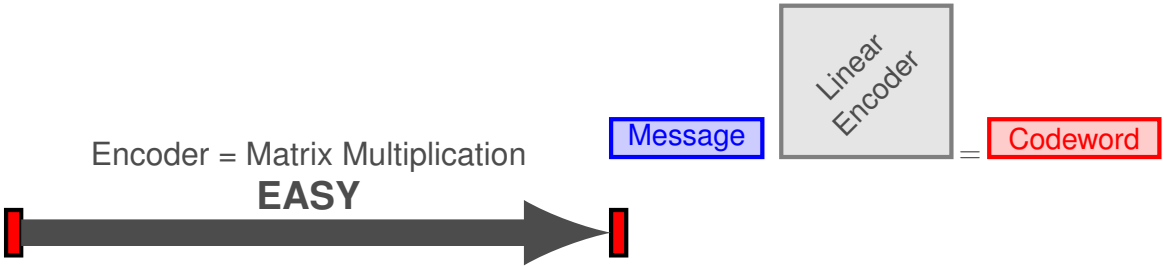
Coding Theory vs. Cryptography

Coding Theory

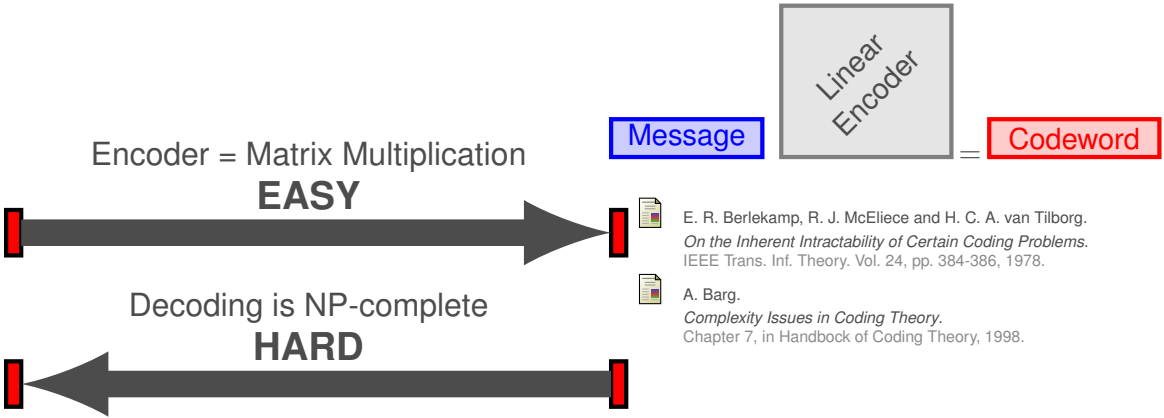
Cryptography

Coding Theory vs. Cryptography

Trapdoor one-way functions - Decoder



Trapdoor one-way functions - Decoder



E. R. Berlekamp, R. J. McEliece and H. C. A. van Tilborg.
On the Inherent Intractability of Certain Coding Problems.
IEEE Trans. Inf. Theory. Vol. 24, pp. 384-386, 1978.

A. Barg.
Complexity Issues in Coding Theory.
Chapter 7, in Handbock of Coding Theory, 1998.

Trapdoor one-way functions - Decoder

Encoder = Matrix Multiplication

EASY

Decoding is NP-complete

HARD

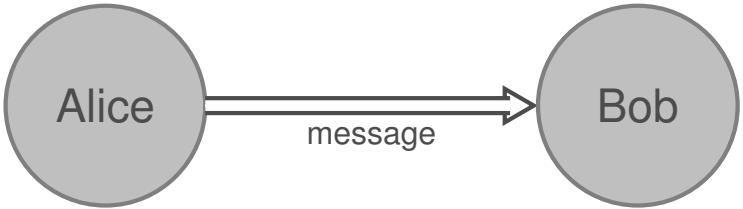
Efficient decoder for certain families of codes

EASY(with TRAPDOOR information)

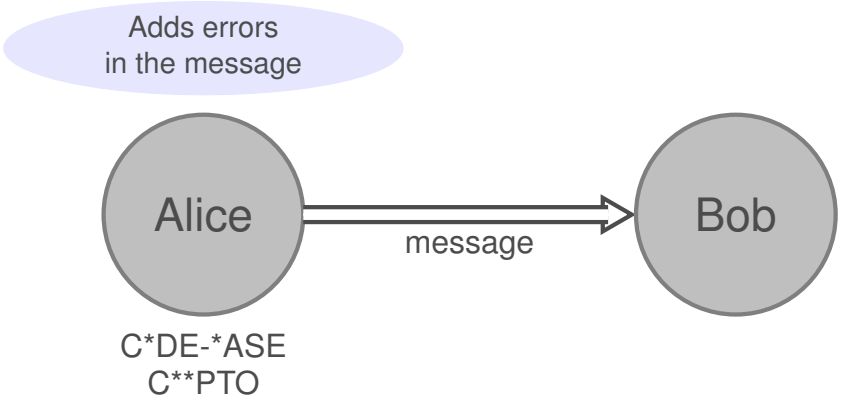
E. R. Berlekamp, R. J. McEliece and H. C. A. van Tilborg.
On the Inherent Intractability of Certain Coding Problems.
IEEE Trans. Inf. Theory. Vol. 24, pp. 384-386, 1978.

A. Barg.
Complexity Issues in Coding Theory.
Chapter 7, in Handbock of Coding Theory, 1998.

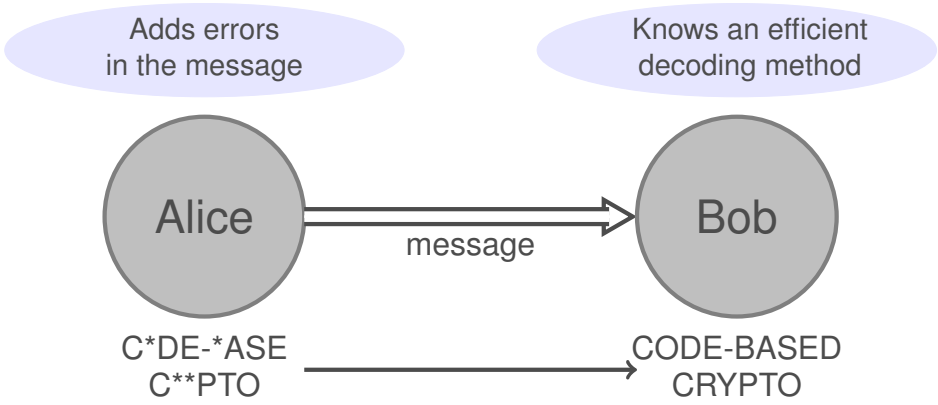
How to use Coding Theory in Cryptography?



How to use Coding Theory in Cryptography?



How to use Coding Theory in Cryptography?



1. Error-Correcting Codes and Cryptography

1. Introduction I - Cryptography
2. Introduction II - Coding Theory
3. **Encoding (Linear Transformation)**
4. Parity Checking
5. Error Correcting Capacity
6. Decoding (A Difficult Problem)
7. Reed-Solomon Codes
8. Goppa Codes
9. McEliece Cryptosystem

Pictures Licenses

- p. 4: Claude Shannon / This image is used courtesy of MIT Museum - more images available at: MIT Museum Collections - People.

- p. 6: Rights Reserved