

Introduction to Functional Programming in *OCaml*

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Week 4 - Sequence 5: Functions on Lists: Folding

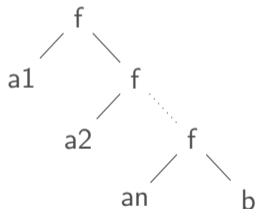


Mapping and Folding

- ▶ Previous sequence: mapping a *unary* function over a list
- ▶ Mapping: All elements considered in isolation
- ▶ Folding: Combining all elements of a list using a *binary* operator
- ▶ Two different ways of folding: fold-left and fold-right

Folding Right

- ▶ Type de `List.fold_right`: `('a -> 'b -> 'b) -> 'a list -> 'b -> 'b`
- ▶ Computing `List.fold_right f [a1; a2; ... an] b`:



Folding Right I

```
let rec fold_right f l b = match l with  
  | [] -> b  
  | h::r -> f h (fold_right f r b);;
```

```
# val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =  
  <fun>
```

```
fold_right (fun x y -> x::y) [1;2;3;4] [];;
```

```
# - : int list = [1; 2; 3; 4]
```

```
fold_right (+) [1;2;3;4] 0;;
```

```
# - : int = 10
```

```
fold_right ( * ) [1;2;3;4] 1;;
```

```
# - : int = 24
```

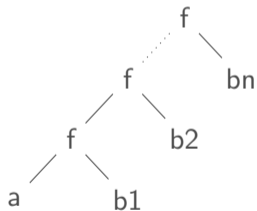
Folding Right II

```
let concat = fold_right (fun x y -> x::y);;  
# val concat : 'a list -> 'a list -> 'a list = <fun>
```

```
concat [1;2;3;4] [5;6;7;8];;  
# - : int list = [1; 2; 3; 4; 5; 6; 7; 8]
```

Folding Left

- ▶ Type de `List.fold_left`: `('a -> 'b -> 'a) -> 'a -> 'b list -> 'a`
- ▶ Computing `List.fold_left f a [b1; b2; ... bn]`



Folding left I

```
(* defined in library as List.fold_left *)
```

```
let rec fold_left f b = function
```

```
  | [] -> b
```

```
  | h::r -> fold_left f (f b h) r;;
```

```
# val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =  
  <fun>
```

```
fold_left (+) 0 [1;2;3;4];;
```

```
# - : int = 10
```

```
let reverse = fold_left (fun x y -> y::x) [];;
```

```
# val reverse : 'a list -> 'a list = <fun>
```

```
reverse [1;2;3;4;5];;
```

```
# - : int list = [5; 4; 3; 2; 1]
```

Case Study: Inner Product of Integer Vectors

- ▶ Inner Product:

$$[x_1; x_2; x_3] * [y_1; y_2; y_3] = [x_1 * y_1 + x_2 * y_2 + x_3 * y_3]$$

- ▶ Computation : first compute list of pair-wise products, then sum up.

Example: Inner Product of Integer Vectors I

```
let product v1 v2 =  
  List.fold_left (+) 0 (List.map2 ( * ) v1 v2);;  
# val product : int list -> int list -> int = <fun>
```

```
product [2;4;6] [1;3;5];;  
# - : int = 44
```

Example: Counting Elements of a List I

```
(* count elements of l satisfying p *)  
let countif p l = List.fold_left  
  (fun counter element -> if p element then counter+1 else counter)  
  0  
  l  
;;  
# val countif : ('a -> bool) -> 'a list -> int = <fun>  
  
countif (function x -> x>0) [3;-17;42;-73;-256];;  
# - : int = 2
```

To Know More

The OCaml Manual:

- ▶ The standard library
 - ▶ `Module List`